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THE HURST STATISTICS OF THE TIME FLOW OF STRUCTURAL DAMAGE OF COMPOSITES AS A MEASURE OF THE EVOLUTION OF A FRACTURE SPOT

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The general thermodynamic theory of the evolution of any system is based on the Prigozhin criterion [1]

$$\frac{d_X P}{dt} \leqslant 0, \tag{1}$$

where P is the entropy production in the system,

$$\frac{d_X P}{dt} = \int\limits_V \sum_\alpha \left(J_\alpha \frac{dX_\alpha}{dt}\right) dV,$$

J_α and X_α are the flows and their associated "forces" (according to Onsager's terminology), and V is the volume of the system. Thus, we have

$$\frac{dP}{dt} = \frac{d_X P}{dt} + \frac{d_J P}{dt} \quad \left[\frac{d_J P}{dt} = \int\limits_V \Sigma\left(X_\alpha \frac{dJ_\alpha}{dt}\right) dV\right]. \tag{2}$$

In the linear Onsager thermodynamics of irreversible processes [1], the relation between the "forces" and flows has the form

$$J_\alpha = L_{\alpha\beta} X_\beta. \tag{3}$$

Summation here is performed over the repeated subscript β, $L_{\alpha\beta} = L_{\beta\alpha}$.

By virtue of relations (2) and (3), we write Eq. (1) as

$$\frac{d_X P}{dt} = \frac{d_J P}{dt} = \frac{1}{2}\frac{dP}{dt} \leqslant 0. \tag{4}$$

Inequality (4) is the Prigozhin theorem on the minimum of entropy production, which follows from the general evolution criterion (1) for linear Onsager systems.

We consider the evolution of a fracture spot in loaded composites by assuming that in the accumulation and rapid propagation of microcracks there is local equilibrium among small volumes of the fracture surface and parts of the volume, because microscopic parts of the system reach equilibrium much earlier than equilibrium is established among them. We can therefore introduce the temperature, chemical potential, and other thermodynamic parameters of the spot.

In this case, the Gibbs equation in the coordinate increments has the form

$$\delta e = T\delta S + \sigma_{ij}\delta\varepsilon_{ij} - 2\gamma\delta\Sigma, \tag{5}$$

where e is the internal energy per unit volume of the spot; σ_{ij} and ε_{ij} are the stress- and strain-tensor components, T is the absolute temperature, S is the entropy per unit spot volume, γ is the effective surface fracture energy, and $\delta\Sigma$ is the increment of the fracture surface at the spot.

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Using (5) and some relations of the theory of elasticity [2], we derive (by analogy with [1]) an expression for the second differential of entropy, which is of importance in the equilibrium theory of irreversible systems [1]:

$$\delta^2 S = -\frac{1}{T}\left[\frac{C_\varepsilon}{T}(\delta T)^2 + K(\delta\theta)^2 + 2\mu\delta\varepsilon_{ij}^{\rm sh}\delta\varepsilon_{ij}^{\rm sh} - 2\frac{\partial\gamma}{\partial\Sigma}(\delta\Sigma)^2\right] < 0. \qquad (6)$$

Here C_ε is the heat capacity under constant strain; K is the bulk elastic modulus, μ is the shear modulus, $\theta = \varepsilon_{ii}$ is the bulk strain, and $\varepsilon_{ij}^{\rm sh}$ is a pure shear strain.

Since in the thermodynamic equilibrium state, the entropy is maximal [1], the condition of stable equilibrium of the spot $\delta^2 S < 0$ in the presence of cracks is satisfied for any small increments of the parameters δT, $\delta\theta$, $\delta\varepsilon_{ij}^{\rm sh}$, and $\delta\Sigma$, only if $\partial\gamma/\partial\Sigma < 0$. This well-known condition for the stable growth of cracks [3] follows, as can be seen from (6), from the fact that entropy must be maximal at the point of stable equilibrium of the fracture spot.

We now analyze the general evolution criterion (1) for the evolution of the fracture spot. By analogy with [1], this criterion is written as

$$-\int\limits_V \frac{1}{T}\left[C_\varepsilon\left(\frac{\partial T}{\partial t}\right)^2 + K\left(\frac{\partial\theta}{\partial t}\right)^2 + 2\mu\frac{\partial\varepsilon_{ij}^{\rm sh}}{\partial t}\frac{\partial\varepsilon_{ij}^{\rm sh}}{\partial t} + 2\gamma\frac{\partial T^{-1}}{\partial t}\frac{\partial\Sigma}{\partial t}\right]dV = \frac{d_X P}{dt} = \int\limits_V \left[W_j\frac{\partial T_{,j}^{-1}}{\partial t}\right]dV \leqslant 0, \qquad (7)$$

where W_j is the jth component of the heat flux, $T_{,j}^{-1} = \partial T^{-1}/\partial x_j$, and x_j is a Cartesian coordinate.

This criterion follows only from the assumption of local spot equilibrium, and does not impose limitations on the values of the first time derivatives of the spot parameters. The general evolution criterion (7) indicates the direction of the evolution of the spot as an irreversible nonequilibrium thermodynamic system. In studies of the spot evolution, it is necessary, however, to know the points of transition of the system from one stationary state to another at which the behavior of the system becomes chaotic (bifurcation points). At these points, the system evolution can further follow qualitatively different paths, depending on small fluctuations of its parameters [1]. This work is devoted to the experimental search for indices of spot evolution.

A method for recording r.f.pulses caused by charge separation on the microcrack sides [4] was used to study the time flows of microcracks upon loading of composites. It is known that crack incubation and rapid propagation in solid dielectrics (including composites) give rise to short r.f. pulses (from $1 \cdot 10^{-6}$ to $100 \cdot 10^{-6}$ sec) with amplitudes of $11.5 \cdot 10^{-3}$ to $600 \cdot 10^{-3}$ V, depending on the sizes of the cracks formed. The r.f. pulses are readily identified by their characteristic bell shape, and their amplitudes are higher by several orders of magnitude than the amplitudes of the pulses produced by other electromagnetic phenomena that accompany the deformation and fracture of dielectrics [4].

The time flows of cracks in composites was studied on a laboratory setup (see [4]) that allows one to record the applied stress, store and photograph the r.f. pulse shape using the screen of an S8-12 two-gun oscillograph, record the total number of r.f. pulses accumulated at time t, and determine their duration and amplitude.

Statistical treatment of the results from the observation of electromagnetic emission was performed as follows. The interval Δt = const (10, 20, 30 sec ...) was fixed. The number of recorded pulses N_i was fixed on each ith interval throughout loading up to the sample's failure. From time T (for $i > 1$), the following characteristics were calculated:

— the average number of pulses

$$\overline{N}_T = \frac{1}{n}\sum_{i=1}^{n} N_i,$$

— the accumulated deviations

$$N_k(T) = \sum_{i=1}^{k}(N_i - \overline{N}_T) \quad (k = 1, \ldots, n),$$

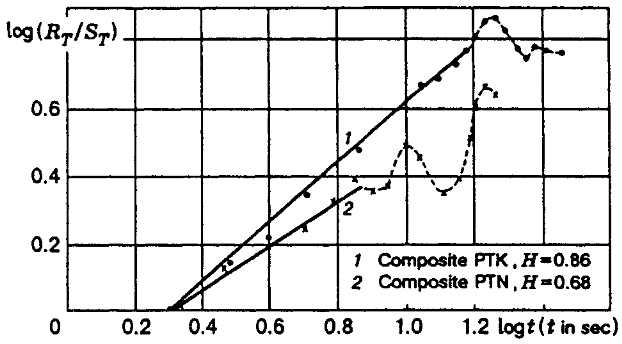

Fig. 1

TABLE 1

Composite	σ_d, MPa	t_d, sec	H	σ_l/σ_d	t_l, sec	Composite	σ_d, MPa	t_d, sec	H	σ_l/σ_d	t_l, sec
PTK	17.2	250	0.67	0.40	100	PTN	7.0	180	0.67	0.42	70
PTK	21.0	400	0.72	0.28	180	PTN	8.0	240	0.69	0.67	165
PTK	5.0	168	0.78	0.40	60	PTN	3.4	280	1.08	0.52	130
PTK	8.9	600	0.86	0.45	320	PTN	4.6	280	0.70	0.35	110

— the scale

$$R_T = \max_k N_k(T) - \min_k N_k(T),$$

— the standard deviation

$$S_T = \left\{ \frac{1}{n} \sum_{i=1}^{n} N_i^2(T) \right\}^{1/2}.$$

The time curve of accumulated deviations for the statistics of R_T/S_T pulses was plotted in the logarithmic coordinates $\log(R_T/S_T) = f(\log t)$.

Investigation of random processes of the type of generalized Brownian motion shows that these processes are well described by the Hurst exponential dependence for the statistics of the normalized scale of accumulated mean deviations [5]:

$$R_T/S_T \sim t^H, \tag{8}$$

where H is the Hurst index and t is time. For random processes with independent increments, the value of the Hurst index is close to 0.5 [5]. An abrupt change in the Hurst index in this case must characterize the chaotic behavior of the system at bifurcation points during the spot evolution.

Figure 1 shows typical results of the study of the time dependences of accumulated deviations for the statistics of electromagnetic-radiation pulses produced by shear loading of PTK and PTN composites up to complete failure.

Analysis of these dependences shows that the statistics of the normalized scale of accumulated mean deviations obeys the Hurst equation (8) up to some moment. Next, a random abrupt change in the index H begins. This can be caused only by transition of the fracture spot formed in the composites to a new qualitative state (the formation of large cracks and their ensembles, which prepare sample failure).

Table 1 gives the stress and time that characterize the instant of the initial transition of a fracture spot to a qualitatively new state during the evolution of the fracture spot for various composites (σ_d is the tensile strength, t_d is the total time before fracture, σ_l/σ_d is the stress at the transition point, and t_l is the instant of the initial transition from the beginning of loading). Table 1 shows that transition can occur both under stresses that are a third of the fracture load and long before complete fracture of the sample.

Thus, the Hurst index H (8) can be a reliable criterion for fracture spot evolution, in particular, at the characteristic points of spot transition to a new state (bifurcation points). In this case, the stationary stage of spot evolution is characterized by a linear region of the logarithmic time curve of accumulated normalized deviations with $H > 0.5$.

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